Subject ELBO

Date

Attendants 22H_Q3

April 21st, 2022

prediction

$p(z|y)$

observation

これが解析的に求めることができないとき

近似

$p(z|y)$ を $p(z)$ で なるべく $p(z|y)$ に近いものにしたい

どれだけ近いかは, $D_{KL}(q(z) \| p(z|y))$

この最小化問題を解くと $p(z|y)$ に近似した $q(z)$ を計算できる

結局 D_{KL} を求めるために,

$p(z|y)$ が必要だけど無理

$\Downarrow$ 別のアプローチ

変化下限

$$\log p(y) = \log\left\{\int p(x,z)\,dz\right\}$$

$$= \log\left\{\int q(z)\frac{p(y,z)}{q(z)}\,dz\right\}$$

$$\geq \int q(z)\log\frac{p(y,z)}{q(z)}\,dz = F(q(z))$$

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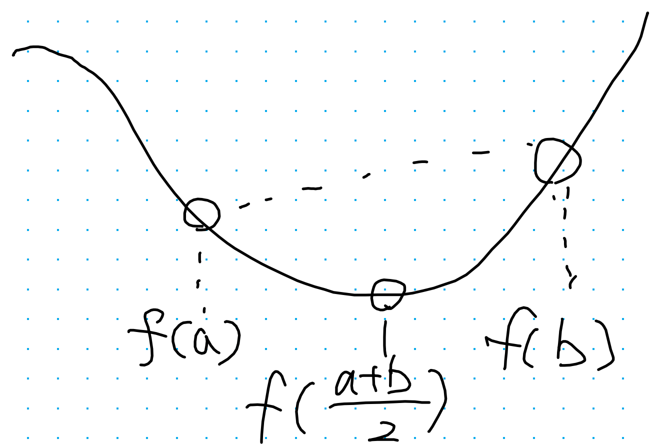
Jensen's Inequality

$$\sum_i \lambda_i f(x_i) \geq f\left(\sum_i \lambda_i x_i\right)$$



So

$$\log\left[\int q(z)\frac{p(y,z)}{q(z)}dz\right]$$

$$\geq \int q(z)\log\frac{p(y,z)}{q(z)}dz = F(q(x))$$

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$$\log p(y) - F[q(z)]$$

$$= \int q(z) \log p(y)\, dz - \int q(z) \log \frac{p(y,z)}{q(z)}\, dz$$

$$= \int q(z) \log \frac{p(y)\, q(z)}{p(y,z)}\, dz$$

$$= \int q(z) \log \frac{q(z)}{p(z|y)}\, dz$$

$$= D_{KL}(q(z) \| p(z|y))$$

So, distance between $p(z|y)$ and $q(z)$

is equal to $\log p(y) - F[q(z)]$

対数周辺尤度 = 変分下限(最大値)の時，KLは最小

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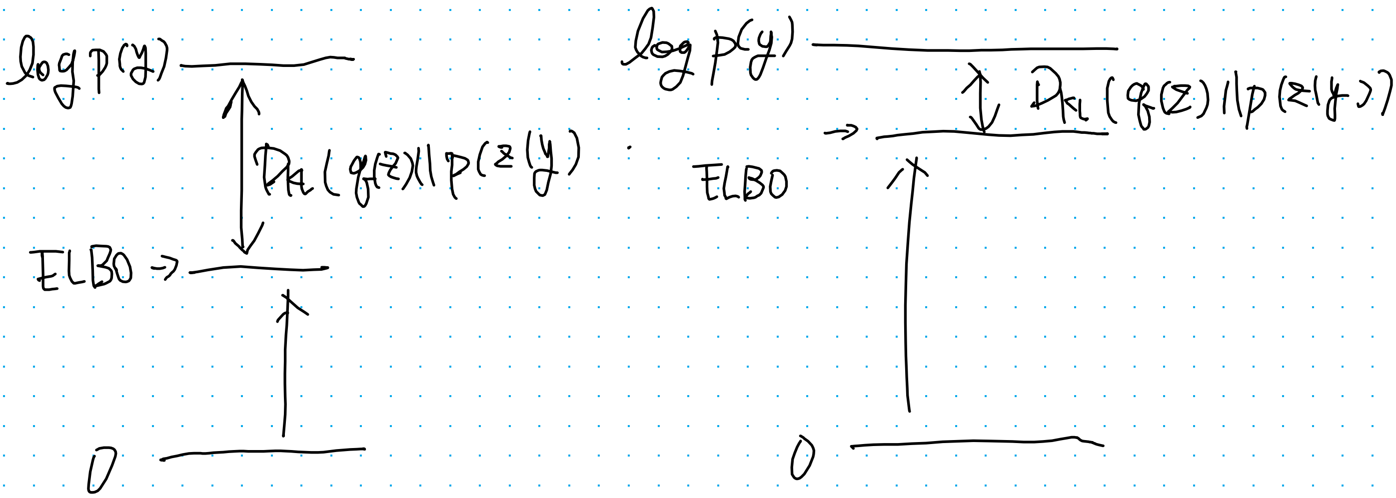

so if we want to approximate $p(z|y)$ by using $q(z)$

we just maximize ELBO : $F[q(z)]$

$$q(z) = \underset{q}{\operatorname{argmax}} F[q(z)]$$

$$= \underset{q}{\operatorname{argmax}} \int q(z) \log \frac{p(x,y)}{q(z)} dx$$

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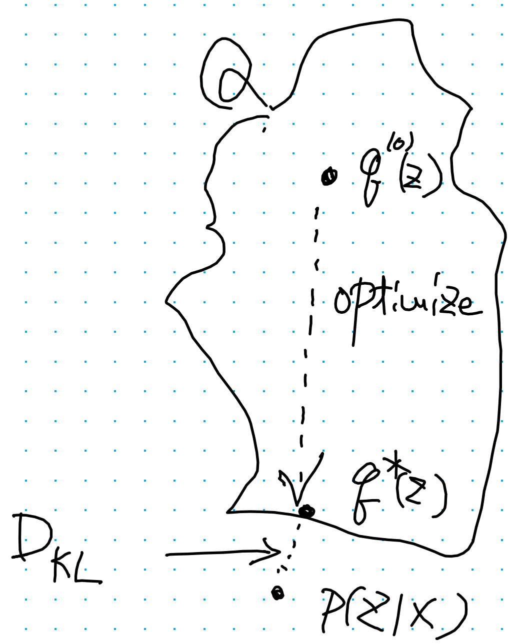
interested in the posterior distribution

$p(z|x)$ $\quad$ x : observation

z : latent variables.

$$p(z|x) = \frac{p(z,x)}{\int p(z,x)\,dz}$$ ← this denominator is not feasible to compute $p(x)$

VI : variational inference



Let $q(z) \approx p(z|x)$

↑

How close ?

Use D_{KL} as metric

$$q^*(z) = \underset{q(z)\in Q}{\arg\min}\; D_{KL}\left(q(z) \,\|\, p(z|x)\right)$$

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$$D_{KL}(q(z) \,\|\, p(z|x))$$

$$= \int_{q(z)} q(z) \log \frac{q(z)}{p(z|x)} \, dz$$

$$= \mathbb{E}_{q(z)}\left[\log \frac{q(z)}{p(z|x)} \right]$$

$$= \mathbb{E}_{q(z)}\left[\log \frac{q(z)}{\frac{p(z,x)}{p(x)}} \right] = \mathbb{E}_{q(z)}\left[\log \frac{q(z)\,p(x)}{p(z,x)} \right]$$

$$= \mathbb{E}_{q(z)}[\log q(z)] + \mathbb{E}_{q(z)}[\log p(x)] - \mathbb{E}_{q(z)}[p(z,x)]$$

$$= \underbrace{\mathbb{E}_{q(z)}[\log q(z)] - \mathbb{E}_{q(z)}[\log p(z,x)]}_{-ELBO(q)} - \mathbb{E}_{q(z)}[p(z,x)]$$

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Since, we cannot compute the desired D_{KL}

$$= \underbrace{\mathbb{E}_{q(z)}[\log q(z)] - \mathbb{E}_{q(z)}[\log p(z,x)]}_{-ELBO(q)} - \mathbb{E}_{q(z)}[p(z,x)]$$

$$ELBO(q) := \mathbb{E}_{q(z)}[\log p(z,x)] - \mathbb{E}_{q(z)}[\log q(z)]$$

then

$$D_{KL}(q(z) \| p(z|x)) = p(x) - ELBO(q)$$

$$p(x) = ELBO(q) + \underbrace{D_{KL}(q(z) \| p(z,x))}_{\geq 0}$$

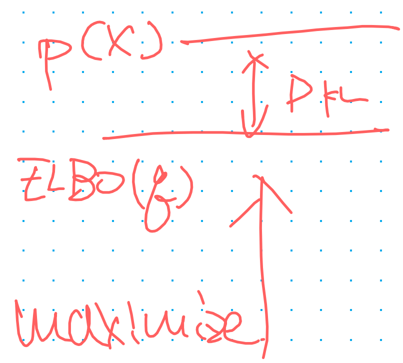
$$\Leftarrow\Rightarrow \quad p(x) \geq ELBO(q)$$

Since $p(x)$ is fixed quantity.

so if we maximize

$ELBO(q)$, then it minimize

$D_{KL}(q(z) \| p(z,x))$.



Subject How to estimate/optimize ELBO Date 8

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In deep generative model

gradient-based VI (such as

Variational Auto Encoder is popular

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