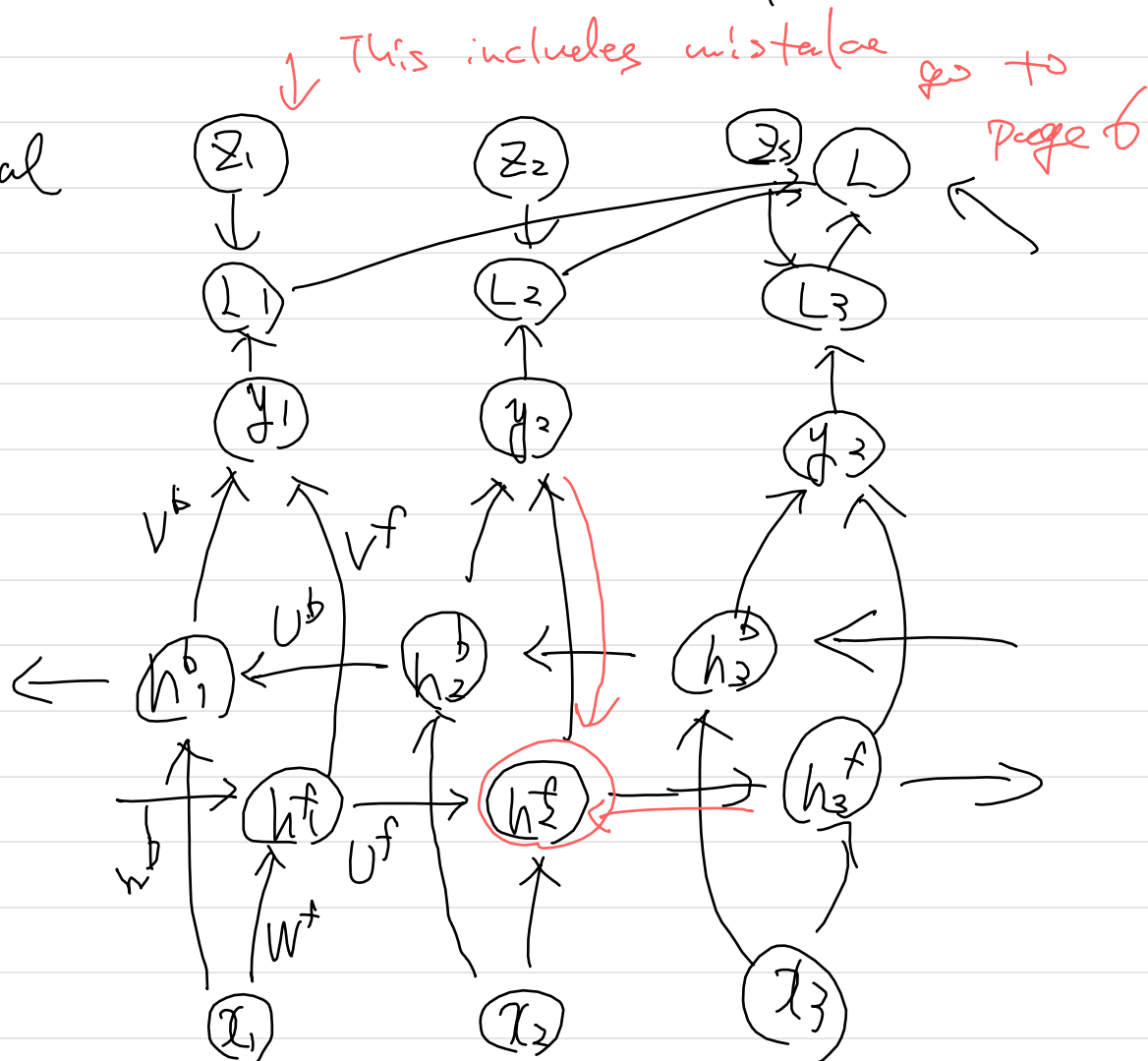


20HL Q4 (Bidirectional RNN) April 20th /

II Draw computational graph



$$\boxed{12} \quad \nabla_{h_t^f} L = \frac{\partial L}{\partial h_t^f} = \frac{\partial L}{\partial h_{t+1}^f} \frac{\partial h_{t+1}^f}{\partial h_t^f} + \frac{\partial L}{\partial L_t} \frac{\partial L_t}{\partial h_t^f}$$

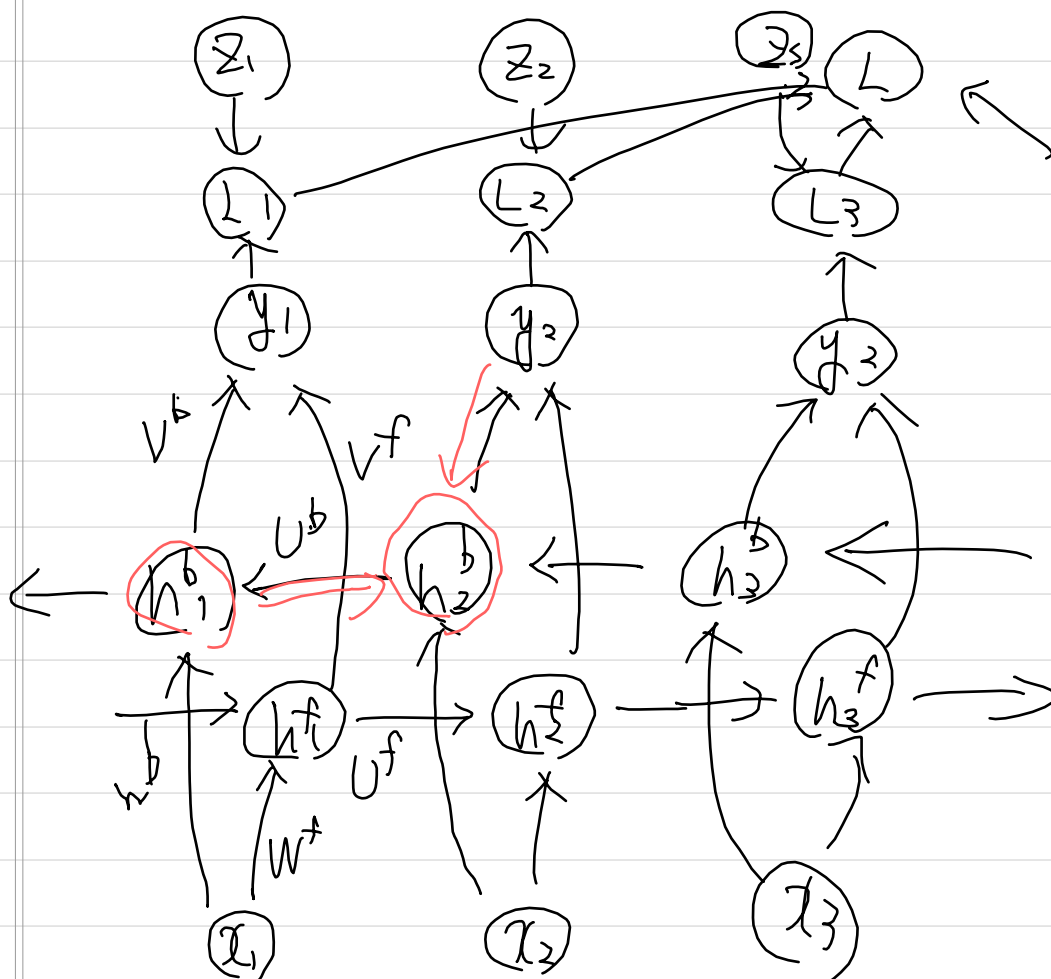
$$\frac{\partial L}{\partial L_t} = 1$$

$$= \nabla_{h_{t+1}^f} L \left(\frac{\partial h_{t+1}^f}{\partial h_t^f} \right) + \nabla_{h_t^f} L_t$$

20H-Q4

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$$\begin{aligned} \nabla_{h_t^f} L_{t+1} &= \frac{\partial L_{t+1}}{\partial h_t^f} = \frac{\partial h_{t+1}^f}{\partial h_t^f} \frac{\partial L_{t+1}}{\partial h_{t+1}^f} \\ &= \left(\frac{\partial h_{t+1}^f}{\partial h_t^f} \right) \nabla_{h_{t+1}^f} L_{t+1} \end{aligned} //$$



$$\begin{aligned} \nabla_{h_t^b} L &= \frac{\partial L}{\partial h_t^b} = \frac{\partial L}{\partial h_{t-1}^b} \frac{\partial h_{t-1}^b}{\partial h_t^b} + \frac{\partial L}{\partial L_t} \frac{\partial L_t}{\partial h_t^b} \\ &= \nabla_{h_{t-1}^b} L \left(\frac{\partial h_{t-1}^b}{\partial h_t^b} \right) + \nabla_{h_t^b} L_t \end{aligned}$$

$$\nabla_{h_t^b} L_t = \frac{\partial L_t}{\partial h_t^b} = \frac{\partial L_t}{\partial y_t} \frac{\partial y_t}{\partial h_t^b} = -y_t \frac{\partial y_t}{\partial h_t^b}$$

20H-Q4

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$$\nabla_{h_t^b} L_t = -2y_t \cdot \frac{\partial y_t}{\partial h_t^b} = -2y_t \cdot V^b$$

$$(y_t = V^f h_{t-1}^f + V^b h_t^b)$$

$$h_t = \sigma(g_t)$$

$$g_t^f = W^f x_t + U^f h_{t-1}^f$$

$$\frac{\partial h_{t+1}^f}{\partial h_t^f} = \frac{\partial h_{t+1}^f}{\partial g_{t+1}^f} \frac{\partial g_{t+1}^f}{\partial h_t^f} \quad (h_{t+1}) \leftarrow (g_t) \leftarrow (h_t)$$

$$= (1 - h_{t+1}^f) h_{t+1}^f \cdot U^f$$

$$\frac{\partial h_{t-1}^b}{\partial h_t^b} = \frac{\partial h_{t-1}^b}{\partial g_{t-1}^b} \frac{\partial g_{t-1}^b}{\partial h_t^b}$$

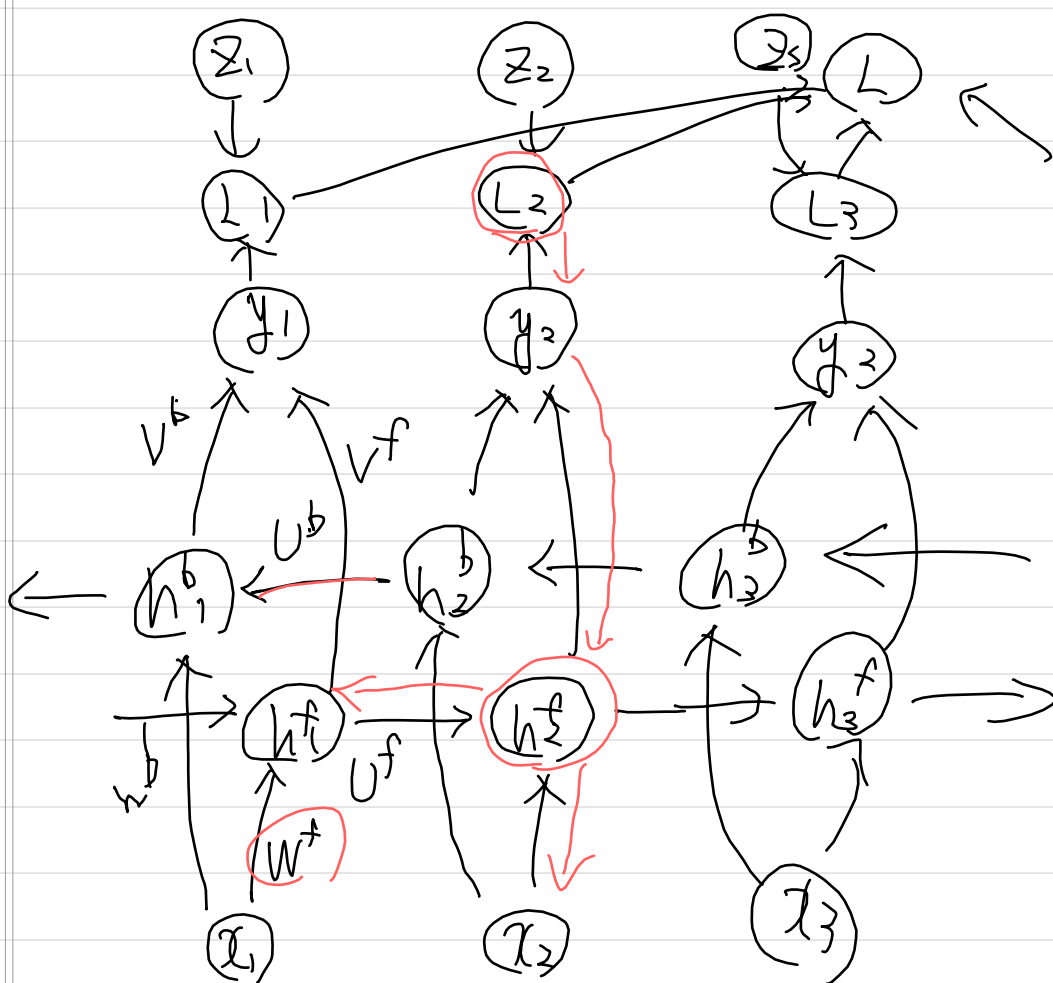
$$g_t^b = W^b x_t + U^b h_{t+1}^b$$

$$= (1 - h_{t-1}^b) h_{t-1}^b \cdot U^b$$

20H_Q4

4

$$\frac{\partial L}{\partial w^f} = \frac{\partial L}{\partial w^f} = \sum_{\tau=1}^T \frac{\partial L_{\tau}}{\partial w^f}$$



then focus

$$\frac{\partial L_{\tau}}{\partial w^f} = \frac{\partial L_{\tau}}{\partial y_{\tau}} \cdot \frac{\partial y_{\tau}}{\partial h_{\tau}^f} \cdot \frac{\partial h_{\tau}^f}{\partial w^f}$$

$$\frac{\partial L_{\tau}}{\partial w^f} = \frac{\partial L_{\tau}}{\partial y_{\tau}} \frac{\partial y_{\tau}}{\partial h_{\tau}^f} \cdot \left(\frac{\partial h_{\tau}^f}{\partial w^f} + \frac{\partial h_{\tau}^f}{\partial h_{\tau-1}^f} \frac{\partial h_{\tau-1}^f}{\partial w^f} \right)$$

20H-Q4

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$$\frac{\partial L^*}{\partial w^f} = \frac{\partial L^*}{\partial y^*} \frac{\partial y^*}{\partial L^{t-1}} \left(\frac{\partial L^{t-1}}{\partial w^f} + \frac{\partial L^{t-1}}{\partial L^{t-2}} \frac{\partial L^{t-2}}{\partial w^f} \right)$$

$$\downarrow$$

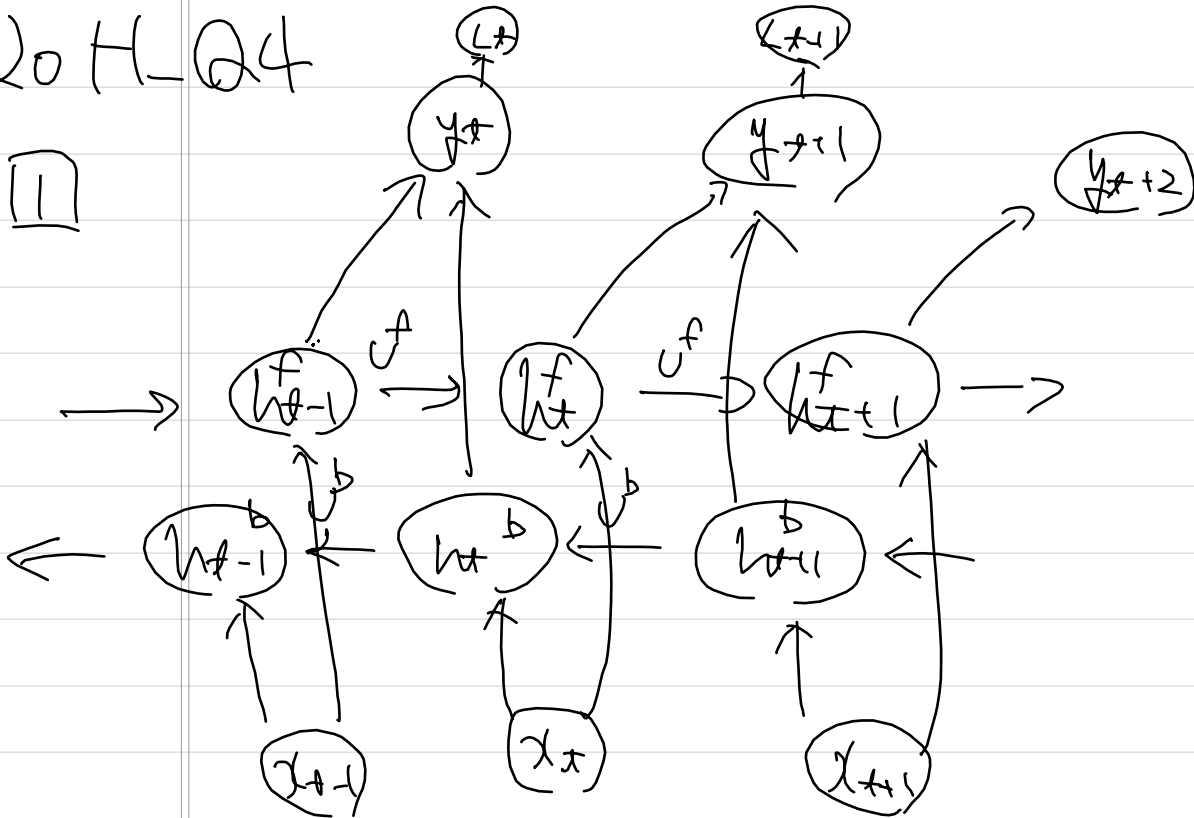
$$-2y^* \cdot \nabla^f \cdot (\quad \dots \quad)$$

to be continued...

20 H-Q4

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(1)



(2)

$$\begin{aligned} \mathbb{D}_{h_t^f}^\top L &= \frac{\partial L}{\partial h_t^f} = \frac{\partial L}{\partial h_{t+1}^f} \frac{\partial h_{t+1}^f}{\partial h_t^f} \\ &\quad + \frac{\partial L}{\partial h_{t+1}^b} \frac{\partial h_{t+1}^b}{\partial h_t^f} \\ &= \underline{\mathbb{D}_{h_{t+1}^b} L_{t+1}} + \left(\frac{\partial h_{t+1}^b}{\partial h_t^f} \right)^\top \mathbb{D}_{h_{t+1}^f}^\top L \end{aligned}$$