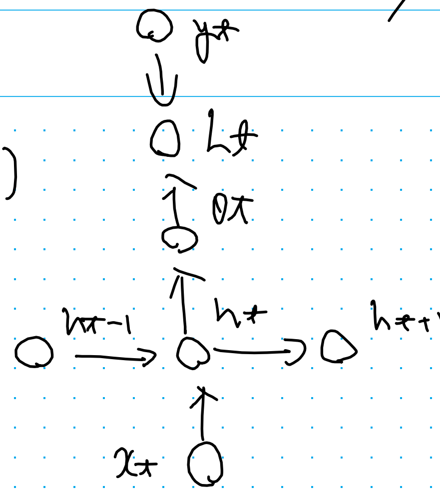
Subject ISH_Q4_9

Date /

Attendants



state $\quad h_t = f(h_{t-1}, x_t, \theta)$

output $\quad o_t = g(h_t, \theta)$

input $\quad x_t$

target $\quad y_t$

loss $\quad L_t = L(o_t, y_t)$, $C = \sum_t L_t$

(a) $\quad \dfrac{\partial C}{\partial L_t} = \dfrac{\partial}{\partial L_t} \sum L_t = 1$

時間的な展開をすると?

then, $\quad \dfrac{\partial C}{\partial \theta} = \dfrac{\partial C}{\partial L_t} \dfrac{\partial L_t}{\partial \theta} = 1 \times \dfrac{\partial L_t}{\partial \theta}$

$$= \frac{\partial L_t}{\partial o_t} \frac{\partial o_t}{\partial \theta} = \underbrace{\frac{\partial L_t}{\partial o_t} \frac{\partial o_t}{\partial h_t}}_{= \frac{\partial L_t}{\partial h_t}} \frac{\partial h_t}{\partial \theta} ?$$

h_tは o_tとh_{t+1}を子ノードとして持つので

$$\frac{\partial L_t}{\partial h_t} = \left(\frac{\partial L_t}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial h_t} + \frac{\partial L_t}{\partial o_t} \frac{\partial o_t}{\partial h_t} \right)$$

$$\frac{\partial C}{\partial \theta} = \frac{\partial L_t}{\partial \theta} = \frac{\partial L_t}{\partial h_t} \frac{\partial h_t}{\partial \theta}$$

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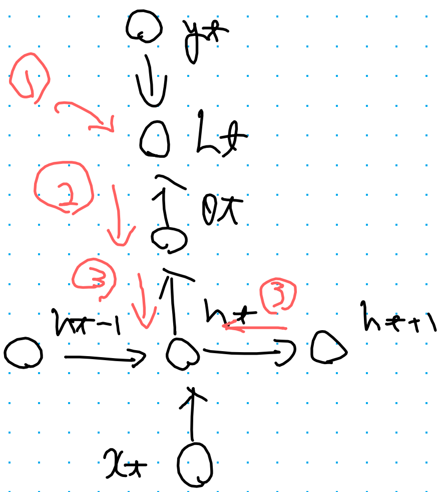
$L_t = L(o_t, y_t)$

$h_t = f(h_{t-1}, x, \theta)$

$o_t = g(h_t, \theta)$



① $\frac{\partial C}{\partial L_t} = 1$, ② $\frac{\partial C}{\partial o_t} = \frac{\partial C}{\partial L_t}\frac{\partial L_t}{\partial o_t}$

$$= \frac{\partial L_t}{\partial o_t} = \nabla_g L$$

③ $$\frac{\partial C}{\partial h_t} = \frac{\partial C}{\partial o_t}\frac{\partial o_t}{\partial h_t} + \frac{\partial C}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial h_t}$$

$$= \frac{\partial C}{\partial L_t}\frac{\partial L_t}{\partial o_t}\frac{\partial o_t}{\partial h_t} + \frac{\partial C}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial h_t}$$

$$= \frac{\partial L_t}{\partial o_t}\frac{\partial o_t}{\partial h_t} + \frac{\partial L_{t+1}}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial h_t}$$

$$= \nabla_g L_t \nabla_f g + \ \cdots \ ?$$

$$\frac{\partial C}{\partial h_t} = \left(\frac{\partial C}{\partial h_t}\right)_{o_t} + \left(\frac{\partial C}{\partial h_t}\right)_{h_{t+1}}$$

$$= \frac{\partial C}{\partial L_t}\frac{\partial L_t}{\partial o_t}\frac{\partial o_t}{\partial h_t} + \underbrace{\frac{\partial C}{\partial L_{t+1}}\frac{\partial L_{t+1}}{\partial o_{t+1}}\frac{\partial o_{t+1}}{\partial h_{t+1}}}\frac{\partial h_{t+1}}{\partial h_t}$$

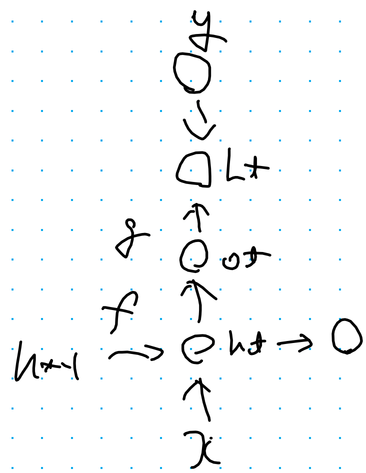
$$= 1 \times \nabla_{g_t} L_t \nabla_{f_t} g_t + 1 \times \nabla_{g_t} L_{t+1} \nabla_{f_{t+1}} g_{t+1} \times \frac{\partial h_{t+1}}{\partial h_t}$$

$$= \nabla_{g_t} L_t \nabla_{f_t} g_t + \frac{\partial C}{\partial L_{t+1}}\frac{\partial h_{t+1}}{\partial h_t}$$

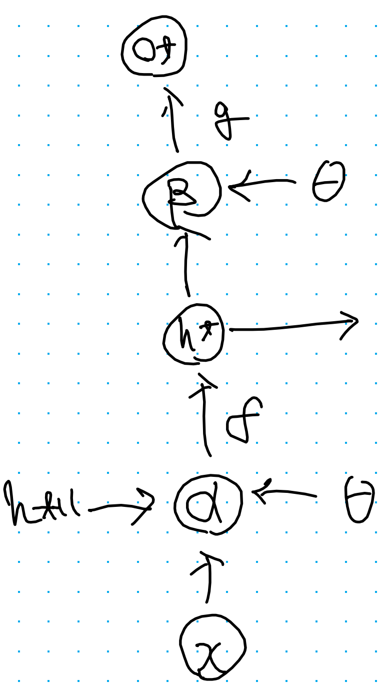
$$\frac{\partial C}{\partial h_t} = \nabla_{g_t} L_t \nabla_{f_t} g_t + \frac{\partial C}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial h_t}$$

$$= \nabla_{g_t} L_t \nabla_{f_t} g_t + \frac{\partial h_{t+1}}{\partial h_t} \left[\nabla_{g_{t+1}} L_{t+1} \nabla_{f_{t+1}} g_{t+1} + \frac{\partial C}{\partial h_{t+2}} \frac{\partial h_{t+2}}{\partial h_{t+1}} \right]$$

↳ recurrence



$$\frac{\partial C}{\partial \theta} = \frac{\partial L_1}{\partial \theta} + \cdots \frac{\partial L_t}{\partial \theta} + \cdots + \frac{\partial L_T}{\partial \theta}$$



$$\frac{\partial L_t}{\partial \theta} = \frac{\partial L_t}{\partial o_t} \frac{\partial o_t}{\partial \theta}$$

h_tもθに依存している.

$$= \frac{\partial L_t}{\partial o_t} \left\{ \frac{\partial}{\partial \theta} g(h_t, \theta) \times \frac{\partial h_t}{\partial \theta} \right\}$$

$$= \frac{\partial L_t}{\partial o_t} \frac{\partial g}{\partial \theta} \frac{\partial}{\partial \theta} f(h_{t-1}, x, \theta)$$

h_{t-1}もθに依存

Now, we assume

$$O_t = g(h_t, \theta) = g(z)$$

$$z = e(h_t, \theta)$$

$$\frac{\partial L_t}{\partial \theta} = \frac{\partial L_t}{\partial O_t}\frac{\partial O_t}{\partial \theta} = \frac{\partial L_t}{\partial O_t}\frac{\partial}{\partial \theta} g(h_t(\theta), \theta)$$

$$= D_{g_t} L_t$$

e.g. $f(x) = x^2$

$z = 4x + 3$

$$\frac{\partial f(z)}{\partial x} = \frac{\partial f}{\partial z} \times \frac{\partial z}{\partial x}$$

$$= 2z \times 4$$

$$= 32x + 24$$

$f(z) = 16x^2 + 9 + 24x$

$$\frac{\partial f(z)}{\partial x} = 32x + 24$$

e.g.

$$\cancel{f(\theta) = \theta \times h(\theta) + \theta^2}$$

$$\cancel{h(\theta) = 3\theta}$$

$$\cancel{\frac{\partial f(\theta)}{\partial \theta} = 2\theta + \theta' h(\theta) + \theta \times \frac{\partial h(\theta)}{\partial \theta}}$$

$$\cancel{= \quad - \quad 3}$$

$$\frac{\partial f}{\partial h}\frac{\partial h}{\partial \theta} = \theta \times 3 = 3\theta$$

$$\boxed{\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial \theta} + \quad = 2\theta + 3\theta}$$

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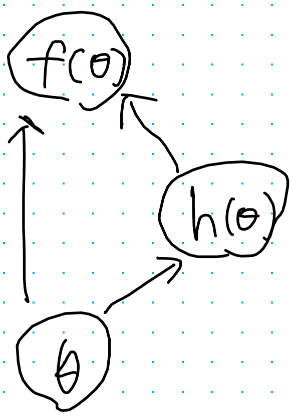
e.g. $f(\theta) = \theta \times h(\theta) + \theta^2$

$h(\theta) = 3\theta$



so, $$\frac{\partial f(\theta)}{\partial \theta} = \left(\frac{\partial f(\theta)}{\partial \theta}\right)_{h(\theta)} + \left(\frac{\partial f(\theta)}{\partial \theta}\right)_{\theta}$$

$$= \frac{\partial f(\theta)}{\partial h(\theta)} \frac{\partial h(\theta)}{\partial \theta} + \frac{\partial f(\theta)}{\partial \theta}$$

$$= \theta \times 3 + 2\theta + h(\theta)$$

$$= 5\theta + 3\theta = 8\theta$$

$f(\theta) = 3\theta^2 + \theta^2 = 4\theta^2$

$\frac{\partial f(\theta)}{\partial \theta} = 8\theta$

一致.

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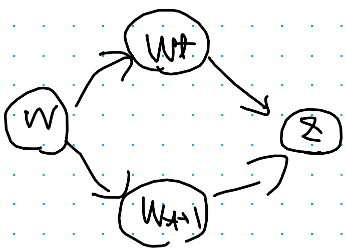

$$\frac{\partial z}{\partial w} = \frac{\partial z}{\partial w_{t+1}}\left(\frac{\partial w_{t+1}}{\partial w}\right) + \frac{\partial z}{\partial w_t}\left(\frac{\partial w_t}{\partial w}\right)$$

いらない？

$\boxed{\left(\frac{\partial L_t}{\partial h_t}\right)_{O_t} \leftarrow \quad h_t \rightarrow O_t \rightarrow L_t \text{ という経路で示す}}$

$$\left(\frac{\partial L_t}{\partial h_t}\right)_{O_t} = \frac{\partial L_t}{\partial O_t}\frac{\partial O_t}{\partial h_t}$$

$$\frac{\partial L_t}{\partial h_t} = \left(\frac{\partial L}{\partial h_t}\right)_{O_t} + \left(\frac{\partial L}{\partial h_t}\right)_{h_{t+1}}$$