

$$\frac{\partial L_t}{\partial \theta} = \frac{\partial L_t}{\partial x_t} \frac{\partial x_t}{\partial \theta}$$

$$= \frac{\partial L_t}{\partial x_t} \left\{ \frac{\partial x_t}{\partial \theta} + \frac{\partial x_t}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \theta} \right\}$$

$$= \frac{\partial L_t}{\partial x_t} \left\{ \frac{\partial x_t}{\partial \theta} + \frac{\partial x_t}{\partial h_{t+1}} \left\{ \frac{\partial h_{t+1}}{\partial \theta} + \frac{\partial h_{t+1}}{\partial h_{t+2}} \frac{\partial h_{t+2}}{\partial \theta} \right\} \right\}$$

$$= \frac{\partial L_t}{\partial x_t} \left\{ \frac{\partial x_t}{\partial \theta} + \frac{\partial x_t}{\partial h_{t+1}} \left\{ \frac{\partial h_{t+1}}{\partial \theta} + \frac{\partial h_{t+1}}{\partial h_{t+2}} \left\{ \frac{\partial h_{t+2}}{\partial \theta} + \frac{\partial h_{t+2}}{\partial h_{t+3}} \frac{\partial h_{t+3}}{\partial \theta} \right\} \right\} \right\}$$

this recurrence has

$$T = C \prod_{i=1}^n \left(\frac{\partial h_{n-1}}{\partial h_n} \right) \quad \text{where } C \text{ is constant}$$

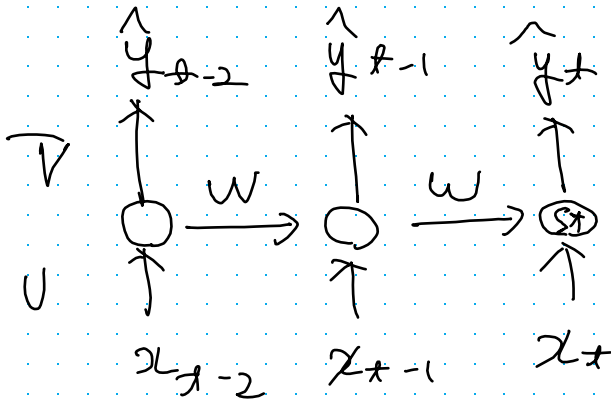
when $n \rightarrow \infty$

$$\text{if } \left(\frac{\partial h_{n-1}}{\partial h_n} \right) > 1 \quad \frac{\partial L_t}{\partial \theta} \rightarrow \infty$$

$$\text{if } \left(\frac{\partial h_{n-1}}{\partial h_n} \right) < 1 \quad \frac{\partial L_t}{\partial \theta} \rightarrow 0$$

so, prevent gradient explosion and

vanishing gradient, $\frac{\partial h_{n-1}}{\partial h_n}$ should be 1



$$s_t = \sigma(Ux_t + Ws_{t-1})$$

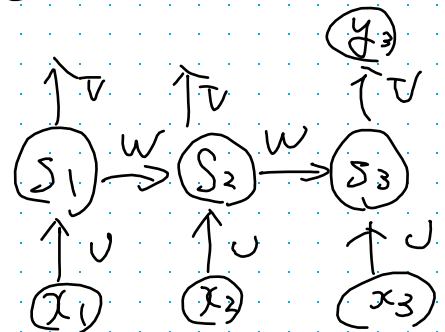
$$y_t = V s_t$$

$$\begin{aligned} \frac{\partial s_t}{\partial s_{t-1}} &= \frac{\partial \sigma(Ux_t + Ws_{t-1})}{\partial s_{t-1}} \\ &= \sigma' \times (Ux_t + Ws_{t-1}) \\ &= \sigma' \times W \end{aligned}$$

$$\frac{\partial E}{\partial w} = \sum_{t=1}^T \frac{\partial E_t}{\partial w}$$

$$\frac{\partial E_t}{\partial w} = \sum_{k=1}^t \frac{\partial E_t}{\partial s_k} \frac{\partial s_k}{\partial s_k} \frac{\partial s_k}{\partial w}$$

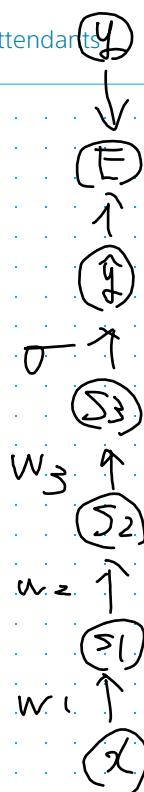
$$\frac{\partial s_t}{\partial s_k} = \prod_{i=k+1}^t \frac{\partial s_i}{\partial s_{i-1}}$$



e.g. $\frac{\partial E_3}{\partial w} = \frac{\partial E_3}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial w}$

深層DNNと同じ状況になっている

Wを深層部分と浅層部分で共有するので $\frac{\partial s_t}{\partial s_{t-1}} = \sigma'(Ux_t + Ws_{t-1})W$
 消失点だけしか発散せず
 pが1より小さいと発散 or 消失



$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial s_3} \frac{\partial s_3}{\partial w_3}$$

$$s_3 = \sigma(w_3 s_2 + b)$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial w_2}$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial s_3} \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial w_1}$$

$$\sigma' w_2$$

$$\sigma' w_1$$

$$\left[\frac{\partial}{\partial s_2} \sigma(w_3 s_2 + b) \right] = \sigma' w_3$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial s_3} \times (\sigma')^3 \times w_3 w_2 w_1$$

$$\boxed{(\sigma' w_3)^3 \times w_3 w_2 w_1}$$