

$$P_{\text{model}}(x; \theta) \rightarrow P_{\text{model}}(x)$$

brief expression

$X = (x_1, \dots, x_n)$ is drawn iid from

a discrete distribution with PMF

$$P_{\text{model}}(x; \theta^*)$$

~~確率質量関数~~

probability mass function

empirical PMF on X as $\hat{p}_{\text{data}}(z)$

$$\hat{p}_{\text{data}}(x) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}(x = x_i),$$

$$f(x) = 1 \quad \text{if } x = 0$$

$$f(x) = 0 \quad \text{otherwise}$$

(a) log likelihood objective

Let
 $g(P_{\text{model}}(x) | \hat{P}_{\text{data}}(x))$ be
the probability of $P_{\text{model}}(x)$
when $\hat{P}_{\text{data}}(x)$ is given.

$P_{\text{model}}(x; \theta)$ can be considered

$$P_{\text{model}}(x | \theta)$$

log likelihood

$$\log P_{\text{model}}(x | \theta)$$

$$D_{KL}(\hat{p}_{data}(x) \parallel p_{model}(x; \theta))$$

$$= \hat{p} \log \left(\frac{\hat{p}}{p} \right) = \hat{p} \log \hat{p} - \hat{p} \log p \geq 0$$

$\hat{p} \log \hat{p}$ は θ に依存しない

つまり $-\hat{p} \log p \geq 0$ の最小化

$\Leftrightarrow \hat{p} \log p$ の最大化

$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \quad \frac{1}{n} \sum_{i=1}^n f(x_i) \log p_{model}(x_i; \theta)$

$\Downarrow ?$

$$= \mathbb{E}_{p_{data}(x)} \left[\log p_{model}(x; \theta) \right]$$