6 (b)

$p(y_i | x_i)$

$$y_i = f_\theta(x_i) + \varepsilon_i = q$$

Gaussian Distribution

$$y_i \sim \underbrace{\mathcal{N}(f_\theta(x_i), \sigma^2)}$$

$p(y|x, \sigma, f_\theta)$

$$= \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(y_i - f_\theta(x_i))^2}{2\sigma^2}\right)$$

6 (c) log likelihood

p is true distribution

$$D_{KL}(p \| q) = \int p \log p - \underbrace{p \log q}_{min}$$

$$\arg\max \; \mathbb{E}_p[\log q]$$

$$= \mathbb{E}_p\left[\log \frac{1}{\sqrt{2\pi}\sigma} + \sigma \log 2 - (y_i - f_\theta(x_i))^2\right]$$

$$\text{So } \hat{f}_\theta = \underset{f_\theta}{\arg\max} \sum_{i=1}^{N} -\left(y_i - f_\theta(x_i)\right)^2$$

$$= \underbrace{\underset{f_\theta}{\arg\min} \sum_{i=1}^{N} \left\{y_i - f_\theta(x_i)\right\}^2}$$

6 (d) Mean Square Error ↑ equivalent.

assume linear function.

6 (e) $f_\theta(x) = wx$

$$\frac{\partial \mathcal{L}}{\partial w} = 2(y_i - wx) \times (wx)'$$

$$0 = 2(\bar{y} - w\bar{x})\bar{x}$$

$$\Longleftrightarrow \quad yx - wx^2 = 0$$

$$w = \sum_{i=1}^{N} \frac{y_i}{x_i}$$