17A_Q4-3

fに関する時間微分に関して

Jacobian $\frac{\partial h_t}{\partial h_\tau}$ を 合成関数 と Chain Rule を使って書き下せ

($\tau < t$)

ただし 全ての行列は同じ eigen-vector

$\frac{\partial h_t}{\partial h_\tau}$ の固有値はどうなる?

$$\frac{\partial h_t}{\partial h_\tau} = \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} \cdots \frac{\partial h_{\tau+1}}{\partial h_\tau}$$

$$\frac{\partial f(h_{t-1}, x_{t-1}, \theta)}{\partial f(h_{t-2}, x_{t-2}, \theta)}$$

$t=4$
$\tau=1$ ならば

$\frac{\partial h_4}{\partial h_3} \frac{\partial h_3}{\partial h_2} \frac{\partial h_2}{\partial h_1}$ の3つ

この行列も eigenvector V を持つ
J

固有値

$J_t = V \Lambda_t V^T$ とすると　$V = V^T = V^{-1}$
$\Leftrightarrow VV^T = I$

$$\frac{\partial h_t}{\partial h_\tau} = J_t \cdot J_{t-1} \cdot \cdots J_{\tau+1}$$

$$= V \Lambda_t \Lambda_{t-1} \cdots \Lambda_{\tau+1} V^T$$

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$h_t = f(h_{t-1}, x, \theta)$

$$\frac{\partial h_t}{\partial h_\tau} = \frac{\partial}{\partial h_\tau} f(h_{t-1}, x, \theta)$$

5/8 page (memo)　fをlinearに仮定?

2/3 page Answer

$$\frac{\partial L}{\partial h_t} = \left(\frac{\partial h_{t+1}}{\partial h_t}\right)^T (\nabla_{h_{t+1}} L) + \left(\frac{\partial o_t}{\partial h_t}\right)^T \nabla_{o_t} L$$

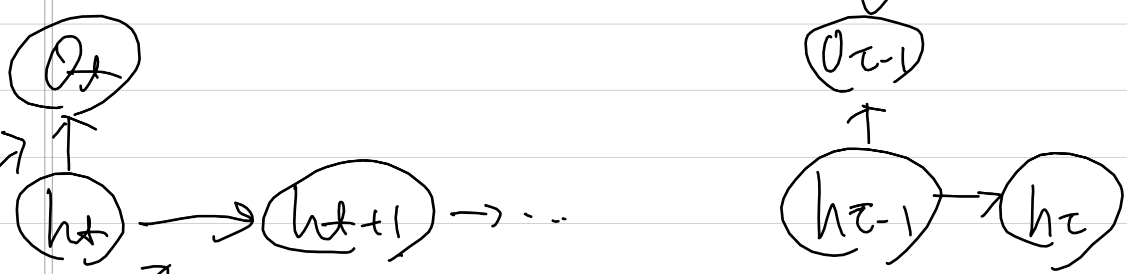
を理解しないと進まない

h_tは　o_tとh_{t+1}をノードとして持つ

$$\frac{\partial L}{\partial h_t} = \underbrace{\frac{\partial h_{t+1}}{\partial h_t} \frac{\partial L}{\partial h_{t+1}} + \frac{\partial o_t}{\partial h_t} \frac{\partial L}{\partial o_t}}_{\text{計算グラフより}}$$

この部分を　$t = t, t+1, \ldots$ としていく.

$$\frac{\partial h_t}{\partial h_\tau} = \frac{\partial h_t}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial h_{t+2}} \cdots \frac{\partial h_{\tau-1}}{\partial h_\tau}$$



$O_t = g(h_t, \theta)$

$h_{t+1} = f(h_t, x_t, \theta)$

$$\frac{\partial h_t}{\partial h_{t+1}} = \frac{\partial f}{\partial h_{t+1}}$$

$t < \tau$ じゃなくて

$\tau < t$ かな?

$$\frac{\partial h_t}{\partial h_\tau} = \frac{\partial h_t}{\partial h_{t-1}} \cdot \frac{\partial h_{t-1}}{\partial h_{t-2}} \cdots \frac{\partial h_1}{\partial h_0}$$

writedown matrix of Jacobian $\frac{\partial h_t}{\partial h_\tau}$

$$\frac{\partial h_t}{\partial h_\tau} = \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} \cdots \frac{\partial h_{\tau+1}}{\partial h_\tau}$$

$$\longrightarrow \begin{pmatrix} \frac{\partial f(h_{t-1}, x, \theta)}{\partial h_{t-1}} & \cdots & \frac{\partial f(h_{t-1}, x, \theta)}{\partial h_\tau} \\ \vdots & & \\ \frac{\partial f(h_\tau, x, \theta)}{\partial h_{t-1}} & \cdots & \frac{\partial f(h_\tau, x, \theta)}{\partial h_\tau} \end{pmatrix}$$

(Jが変化しない(時間に対して)と仮定する)

↳ これは次のQ4で使う.