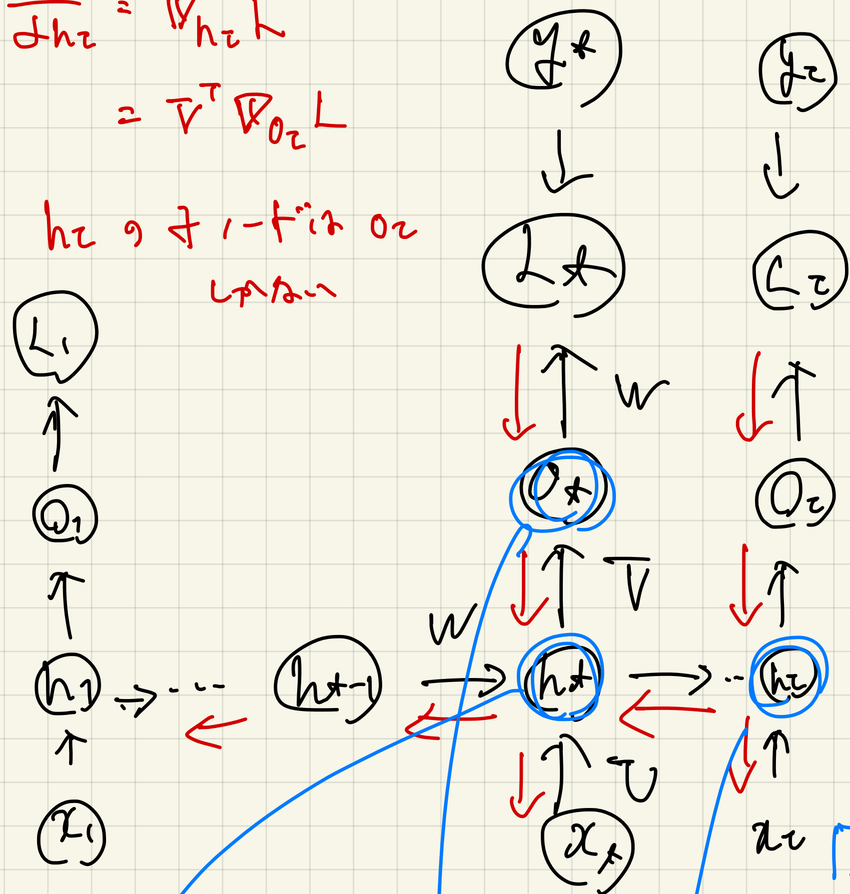
$$\frac{dL}{dh_\tau} = \nabla_{h_\tau} L = V^T \nabla_{o_\tau} L$$

h_τ の子ノードは o_τ しかない



$$\frac{dL}{dL_t} = 1$$

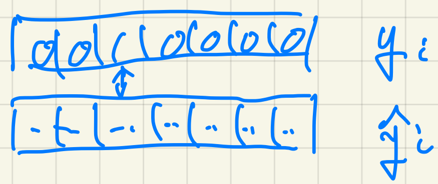
$$\frac{dL}{do_{t,i}} = \frac{dL}{dL_t}\frac{dL_t}{do_{t,i}} = \hat{y}_t - 1_{i,y_t}$$

$$L_t = L(o_t, y_t)$$

$$o_t = g(h_t, \theta)$$
$$= c + V h_t$$

$$h_t = f(h_{t-1}, x_t, \theta)$$
$$= \tanh(a_t)$$

$$a_t = b + W h_{t-1} + U x_t$$



assume $L_t = \frac{1}{2}(y_i - \hat{y}_i)^2$

$$L_t' = \frac{1}{2}2(y_i - \hat{y}_i)$$
$$= (y_i - \hat{y}_i)$$
$$= \hat{y}_i - \underbrace{1_{i,y}}_{\text{one-hot-vector}}$$

h_t は $(t<\tau)$ 子ノードとして o_t と h_{t+1} の両方を持つことに注意する

$$\frac{dL}{dh_t} = \underbrace{\left(\frac{\partial h_{t+1}}{\partial h_t}\right)^T}_{W^T (\tanh')} (\nabla_{h_{t+1}} L) + \underbrace{\left(\frac{\partial o_t}{\partial h_t}\right)^T}_{V^T} (\nabla_{o_t} L)$$

$$\tanh'(a_t) = 1 - \tanh^2(a_t)$$

$$\frac{dL}{dh_t} = \left(\frac{\partial h_{t+1}}{\partial h_t}\right)^T (\nabla_{h_{t+1}} L) + \left(\frac{\partial o_t}{\partial h_t}\right)^T (\nabla_{o_t} L)$$

$$= \left(\frac{\partial}{\partial h_t}\{\tanh(a_{t+1})\}\right)^T (\nabla_{h_{t+1}} L) + V^T \nabla_{o_t} L$$

$$a_{t+1} = b + W h_t + U x_t$$

$$= (1 - \tanh^2(a_{t+1})) \times \frac{\partial}{\partial h_t} a_{t+1} \times \nabla_{h_{t+1}} L + V^T \nabla_{o_t} L$$

$$= \{1 - \tanh^2(a_{t+1})\} \times W^T \times (\nabla_{h_{t+1}} L) + V^T \nabla_{o_t} L$$

$= \mathrm{diag}(1 - (h_{t+1})^2)$ は要素 $1 - (h_{i,t+1})^2$ を持つ対角行列

時刻 $t+1$ で隠れユニット i に関連付けられた $\tanh$ のヤコビ行列.

$$\longrightarrow \nabla_{h_t} L = W^T (\nabla_{h_{t+1}} L)\, \mathrm{diag}(1 - (h_{t+1})^2) + V^T (\nabla_{o_t} L)$$

$$\frac{\partial L}{\partial L_t} = 1$$

$$\frac{\partial L}{\partial o_t} = \underbrace{\hat{y}_t - 1_{i,y_t}}$$

$$\frac{\partial L}{\partial h_t} = \text{前ページ}$$

$$a_t = b + W h_{t-1} + U x_t$$

$$h_t = \tanh(a_t)$$

$$o_t = c + V^T h_t$$

$$y_t = \mathrm{softmax}(o_t)$$

$$\frac{\partial L}{\partial c} = \underbrace{\sum_t \left(\frac{\partial o_t}{\partial c}\right)^T}_{\to 1} \frac{\partial L}{\partial o_t}$$

$$\frac{\partial L}{\partial b} = \sum_t \left(\underbrace{\frac{\partial h_t}{\partial b}}\right)^T \frac{\partial L}{\partial h_t}$$

$$\to (1 - \tanh^2(a_t)) \times \underset{\substack{\shortparallel \\ 1}}{(a_t)'}$$

$$\frac{\partial L}{\partial U} = \underbrace{\sum_t \left(\frac{\partial h_t}{\partial U}\right)^T}\frac{\partial L}{\partial h_t}$$

$$\to (1 - \tanh^2(a_t)) \times \underset{\substack{\shortparallel \\ x_t}}{(a_t)'}$$

$$\frac{\partial L}{\partial V} = \underbrace{\sum_t \left(\frac{\partial t}{\partial V}\right)^T}_{\to h_t} \frac{\partial L}{\partial h_t}$$

$$\frac{\partial L}{\partial W} = \underbrace{\sum_t \left(\frac{\partial h_t}{\partial W}\right)^T} \frac{\partial W}{\partial h_t}$$

$$\to (1 - \tanh^2(a_t)) \times \underset{\substack{\shortparallel \\ h_{t-1}}}{(a_t)'}$$