Proof of $\|w_t\|_2^2 = \sum_{i=1}^{n} \lambda_i^{2t} z_0(i)^2$

ここで

$$\|z_t(i)\|^2 = \langle v_i , w_t \rangle \quad (\text{定義より})$$

$z_t(i)$をまとめてz_tとして考える

$$z_t = \begin{pmatrix} z_t(1) \\ \vdots \\ z_t(p) \end{pmatrix} \quad p \times 1$$

$$= \underbrace{\begin{pmatrix} v_1^T \\ \vdots \\ v_p^T \end{pmatrix}}_{p \times p} (w_t)$$

$$= U^T w_t$$

つまり

$$|z_t|_2^2 = |U^T w_t|_2^2$$

$$= w_t^T (U^T U)^T w_t$$

$$= |w_t|_2^2$$

$z_t(i) = \lambda_i^t z_0(i)$ より

$$|z_t|_2^2 = \sum_{i=1}^{n} (z_t(i))^2 = \sum_{i=1}^{n} \lambda_i^{2t} z_0^2(i)$$