17A Q3 - 8

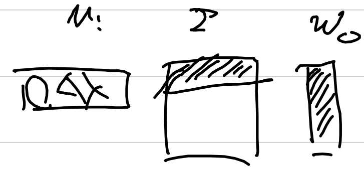
$$z_t(i) = \langle u_i, w_t \rangle$$

$w_t = \Sigma w_{t-1} \ldots$

$$= \langle u_i, \Sigma w_{t-1} \rangle$$ from eq (2)

$$= \langle u_i, \Sigma \times \Sigma w_{t-2} \rangle$$

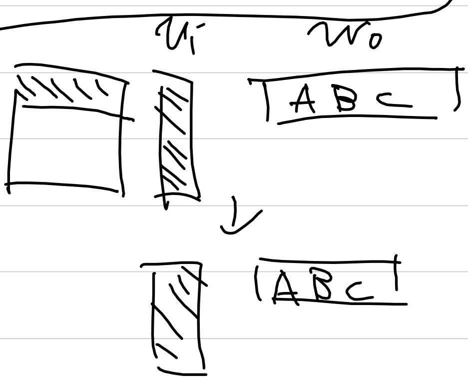
$$= \langle u_i, \Sigma^t w_0 \rangle$$

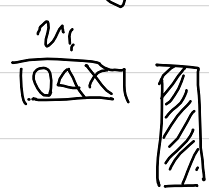


since Σ and Σ^t are symmetric

$\Downarrow$

$$z_t(i) = \langle \underbrace{\Sigma^t u_i} , w_0 \rangle$$





here u_i is eigenvector

$$\Sigma u_i = \lambda_i u_i \quad (i = 1, \ldots, p)$$

$$\Leftrightarrow \underbrace{\Sigma^t u_i} = \lambda_i^t u_i$$

$$z_i(t) = \langle \Sigma^t v_i , w_0 \rangle$$

$$= \langle \underbrace{\lambda_i^t}_{\text{scalar}} v_i , w_0 \rangle$$

$$= \lambda_i^t \underbrace{\langle v_i , w_0 \rangle}_{\text{from definition of } z_0(i)}$$

$$= \lambda_i^t z_0(i)$$