17A Q3-5　　characteristic polynomial

行列Aの固有方程式 = 特性方程式

$\det(A-\lambda E)=0$　を求めればOK

Practice.

$A=\begin{pmatrix} 8 & 1 \\ 4 & 5 \end{pmatrix}$ のとき

$(8-\lambda)(5-\lambda)-4=0$

$\Leftrightarrow 40-13\lambda+\lambda^2-4=0$

$\Leftrightarrow \lambda^2-13\lambda+36=0$

$\Leftrightarrow (\lambda-9)(\lambda-4)=0$

$\lambda=4, 9.$

固有ベクトルは　　$\lambda=4$のとき

$(A-\lambda E)\vec{x}=\vec{0} \Leftrightarrow \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$4x+y=0$

$4x + y = 0$ より

$$x = 1 \times t$$
$$y = -4 \times t$$

tは任意、$t \neq 0$

よって $\vec{x} = t\begin{pmatrix}1\\-4\end{pmatrix}$

次に $\lambda = 9$ のとき

$$\begin{pmatrix}8-9 & 1\\4 & 5-9\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\Longleftrightarrow \begin{cases}-x + y = 0\\4x - 4y = 0\end{cases}$$

$$\Longleftrightarrow x = y$$

$\Longleftrightarrow$ t つまり $\vec{x} = t\begin{pmatrix}1\\1\end{pmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(1-\lambda)(1-\lambda) - 1 = 0$$

$$\Leftrightarrow \quad 1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\Leftrightarrow \quad \lambda(\lambda - 2) = 0 \qquad \lambda = 0, 2$$

In case of $\lambda = 0$

$$(A - \lambda I)\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \quad x + y = 0 \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} t$$

$$\forall t > 0$$

in case of $\lambda = 2$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x = y \qquad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} t$$

$$B = \begin{bmatrix} 1 & -0.7 \\ -0.7 & 1 \end{bmatrix}$$

$$\det(B - \lambda I) = 0 \iff (1-\lambda)^2 - (0.7)^2 = 0$$

$$\iff \{(1-\lambda)+0.7\}\{(1-\lambda)-0.7\} = 0$$

$$\iff \frac{1}{100}(17 - 10\lambda)(3 - 10\lambda) = 0$$

$$\iff (10\lambda - 17)(10\lambda - 3) = 0$$

$$\lambda = \frac{17}{10}, \frac{3}{10}$$

when $\lambda = \frac{3}{10}$ $\quad (B - \lambda I)\vec{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\iff \begin{pmatrix} \frac{7}{10} & -\frac{7}{10} \\ -\frac{7}{10} & \frac{7}{10} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x = y \quad \begin{pmatrix} x \\ y \end{pmatrix} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

when $\lambda = \frac{17}{10}$

$$\begin{pmatrix} -\frac{7}{10} & -\frac{7}{10} \\ -\frac{7}{10} & -\frac{7}{10} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x + y = 0 \quad \begin{pmatrix} x \\ y \end{pmatrix} = t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(C - \lambda I) = 0 \iff (1-\lambda)^2 = 0$

$\lambda = 1$

in case $\lambda = 1$ $\quad (C - \lambda I)\vec{x} = \vec{0}$

$\iff \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} s \\ t \end{pmatrix} \quad \forall s, t.$

$D = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(D - \lambda I) = 0 \iff (5-\lambda)(1-\lambda) = 0$

$\lambda = 1, 5$

when $\lambda = 1$

$\begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad 4x = 0$

$\begin{pmatrix} x \\ y \end{pmatrix} = t \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

when $\lambda = 5$

$\begin{pmatrix} 0 & 0 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad y = 0 \qquad \begin{pmatrix} x \\ y \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \end{pmatrix}$