

17A_Q3

1. $x_i \in \mathbb{R}^p$, sample $x_1 \dots x_n$

Empirical Centered Covariance is as follows.

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\Sigma = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)(x_i - \mu)^T$$

where $\Sigma \in \mathbb{R}^{p \times p}$

we also describe above like follows

$$\Sigma = \frac{1}{n} \sum_{i=1}^n \left[(x_i - \mu)(x_i - \mu)^T \right]$$

2. Prove Σ is PSD

let $u \in \mathbb{R}^p$, then

$$\begin{aligned} u^T \Sigma u &= u^T \mathbb{E}[(X - \mathbb{E}[X])(X - \mathbb{E}[X])^T] u \\ &= u^T \mathbb{E}[\langle u, X - \mathbb{E}[X] \rangle \langle X - \mathbb{E}[X], u \rangle] \\ &= \mathbb{E}[\langle u, X - \mathbb{E}[X] \rangle^2] \geq 0 \end{aligned}$$

therefore Σ is PSD $\square$