17A Q-3-10

Finite iteration Analysis.

assume. $|z_0(i)| = \frac{1}{\sqrt{p}}$ with probability. 1

$\lambda_1 = \dots = \lambda_p$

from Q3-9

$$Q(z_t) = 1 - \frac{\sum_{i=2}^{n} \lambda_i^{2t} z_0(i)}{\sum_{i=1}^{n} \lambda_i^{2t} z_0(i)}$$

show $\underbrace{Q(z_t) > 1-\varepsilon}$

where $t > \frac{\log\frac{1-\varepsilon}{\varepsilon} + \log(p-1)}{2\log(\lambda_1/\lambda_2)}$

<=> show

approx error < ε

approx error

$$= \frac{\sum_{i=2}^{p} \lambda_i^{2t} z_0^2(i)}{\sum_{i=1}^{p} \lambda_i^{2t} z_0^2(i)}$$

$$= \frac{\sum_{i=2}^{p} \lambda_i^{2t} z_0^2(i)}{\lambda_1^{2t} z_0^2(1) + \sum_{i=2}^{p} \lambda_i^{2t} z_0^2(i)}$$

$|z_0(i)| = \frac{1}{\sqrt{p}}$ then,

$$\text{approx error} = \frac{\frac{1}{p}\sum_{i=2}^{p} \lambda_i^{2t}}{\lambda_1^{2t}\frac{1}{p} + \frac{1}{p}\sum_{i=2}^{p} \lambda_i^{2t}}$$

$$= \frac{\sum_{i=2}^{p} \lambda_i^{2t}}{\lambda_1^{2t} + \sum_{i=2}^{p} \lambda_i^{2t}}$$

$= \lambda_1 = \lambda_2 = \cdots \lambda_n$

$$= \frac{(p-1)\lambda_2^{2t}}{\lambda_1^{2t} + (p-1)\lambda_2^{2t}}$$

... ※

From assumption,

$$\star > \frac{\log\frac{1-\varepsilon}{\varepsilon} + \log(p-1)}{2\log(\lambda_1/\lambda_2)}$$

$$\Leftrightarrow \log\frac{1-\varepsilon}{\varepsilon} + \log(p-1) < 2\star \log\frac{\lambda_1}{\lambda_2}$$

$$\Leftrightarrow \left(\frac{1-\varepsilon}{\varepsilon}\right)(p-1) < \left(\frac{\lambda_1}{\lambda_2}\right)^{2\star}$$

$$\Leftrightarrow (p-1)(1-\varepsilon) < \varepsilon\left(\frac{\lambda_1}{\lambda_2}\right)^{2\star}$$

$$\Leftrightarrow (p-1) - \varepsilon(p-1) < \varepsilon\left(\frac{\lambda_1}{\lambda_2}\right)^{2\star}$$

$$\Leftrightarrow (p-1) < \varepsilon(p-1) + \varepsilon\left(\frac{\lambda_1}{\lambda_2}\right)^{2\star}$$

$$\Leftrightarrow (p-1)\lambda_2^{2\star} < \varepsilon\lambda_1^{2\star} + \varepsilon(p-1)\lambda_2^{2\star}$$

$$\Leftrightarrow \frac{(p-1)\lambda_2^{2\star}}{\lambda_1^{2\star} + (p-1)\lambda_2^{2\star}} < \varepsilon$$

So we showed $\star$ as above